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EXTINCTION OF SPHERICAL DIFFUSION FLAMES: SPALDING'S APPROACH

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NOMENCLATURE

B, transfer number;
b, equation (7d);
C, stoichiometric mass of core species;
D, diffusion coefficient;
G, mass burning velocity through a stoichiometric mixture;
I, stoichiometric mass of species i;
$\dot{m}$, mass injection rate;
p, pressure;
R, volumetric consumption rate of injected fuel;
r, radial coordinate;
t, time;
V, mass of volatiles in solid fuel;
v, velocity component;
Y, mass fraction;
β, coupling function, equation (4b);
ε, flux fraction;
Λ, equation (7c);
ν, stoichiometric parameter, equation (16b);
ν_s, stoichiometric oxygen–fuel mass ratio;
ξ, dimensionless radial coordinate, equation (4a);
ϕ, equation (4c);
ρ, density;
Ψ, dimensionless reaction rate;
$\dot{\omega}$, rate of species generation by chemical reaction.

Subscripts

c, core species;
F, fuel;
i, species i;
0, initial value;
O, oxygen;
s, solid;
st, stoichiometric;
v, volatiles;
w, wall condition;
∞, condition at infinity.

Superscripts

$'''$, per unit volume;
$+$, dimensionless quantity.

1. INTRODUCTION

THE EXTINCTION problem of droplets has been studied in some detail [1–4] and an extinction criterion in a closed form for opposed jet diffusion flame was obtained by Spalding [5], using an approximate analytical technique. Del Notario *et al.* [6] investigated the extinction of spherical and premixed diffusion flames in air. This communication presents a closed form solution for the gas extinction of a spherical diffusion flame. It attempts to link the method of solution of an opposed jet diffusion flame as obtained by Spalding [5] with that of a spherical symmetric non-adiabatic diffusion flame.

2. GOVERNING EQUATIONS

Consider a porous sphere through which a mixture of fuel and inert gas is injected. The conservation equations are

$$\text{Species:} \quad \rho v \frac{\mathrm{d}Y_i}{\mathrm{d}r} = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho r^2 D \frac{\mathrm{d}Y_i}{\mathrm{d}r}\right) + \dot{\omega}_i''' \tag{1}$$

$$\text{Momentum:} \quad p = \text{constant} \tag{2}$$

$$\text{Overall Continuity:} \quad 4\pi\rho v r^2 = \dot{m}. \tag{3}$$

Let the Lewis number equal to unity.

$$\xi = \frac{\dot{m}}{4\pi\rho D r}, \qquad \beta_{ic} = \frac{Y_i}{I} - \frac{Y_c}{C} \tag{4a, b}$$

and

$$\phi_{ic} = \frac{\beta_{ic} - \beta_{ic,\infty}}{\beta_{ic,w} - \beta_{ic,\infty}}. \tag{4c}$$

From equations (1) and (4) for all $i \neq c$

$$\phi_{ic} = \frac{1 - \exp(-\xi)}{1 - \exp(-\xi_w)}. \tag{5}$$

If the fuel is the core species, then $c = F$ and $i = O$. The species conservation equation is first written in terms of ξ as the independent parameter and then ϕ_{OF} is used as a transformation coordinate to yield

$$\frac{d^2 Y_F}{d\phi_{OF}^2} = -\dot{\omega}_F''' \dot{m}^2 \Big/ \left[\left(\frac{d\phi_{OF}}{d\xi} \right)^2 (4\pi)^2 (\rho D)^3 \xi^4 \right]. \tag{6}$$

Let

$$R = -\dot{\omega}_F''', \qquad \Psi = R/R_{max} \tag{7a, b}$$

$$\Lambda = \dot{m}^2 R_{max}/(4\pi)^2(\rho D)^3 \tag{7c}$$

and

$$b = 1 - \phi_{OF}[1 - \exp(-\xi_w)]. \tag{7d}$$

Also, for thermodynamic equilibrium in this system of species

$$(\phi_{OF})_{st} = \frac{1-\exp(-\xi_{st})}{1-\exp(-\xi_w)} = \frac{Y_{O,\infty}/v_s}{Y_{F,w} + Y_{O,\infty}/v_s} \tag{8}$$

and the consumption rate of the fuel in the flame is given by

$$\varepsilon_F = \left(\frac{dY_F}{d\phi_{OF}} \right)_{st} \exp(-\xi_{st})/[1-\exp(-\xi_w)]. \tag{9}$$

Hence from equations (6), (7), (8) and (9) one obtains

$$\varepsilon_F v_s / Y_{O,\infty} = b_{st}/(1-b_{st}) \tag{10}$$

and

$$\frac{d^2 Y_F}{db^2} = \Lambda \Psi / b^2 (\ln b)^4. \tag{11}$$

3. SOLUTIONS

The parameter Λ in equation (11) must be varied to achieve a value of which the only solution for the problem corresponds to zero reaction. If this equation is compared with Spalding's equation for opposed jet diffusion flame [5], it is seen that the two equations are similar except for the function containing b.

Equation (11) is integrated to obtain

$$1/2 \left\{ \left(\frac{dY_F}{db} \right)^2 \right\}_{b=1}^{b=\exp(-\xi_w)} = \Lambda \int_0^{Y_{F,w}} \frac{\Psi}{b^2 (\ln b)^4} dY_F. \tag{12}$$

The technique of approximating the solution is similar to the treatment given in [5]. The fuel mass fraction Y_F is plotted against the quantity b with Ψ as a parameter and the resulting diagram is shown in Fig. 1. The summit of the Ψ mountain is very close to $b = b_{st}$ for a high activation energy system. The left hand side of equation (12) decreases monotonically from a maximum value for the case of complete thermodynamic equilibrium to a value near zero for the mixing problem. During the same process, the right hand side of the equation goes through a maximum. Hence, when Λ falls below a minimum value the only solution where the equality of equation (12) holds is the case of zero reaction. For this case then,

$$\frac{\dot{m}^2 R_{max}}{(4\pi)^2(\rho D)^3} = \Lambda_{min} \approx 1/2 \left\{ \left[\frac{Y_{F,w}}{b_{st} - \exp(-\xi_w)} \right]^2 b_{st}^2 \right\} \times \frac{(\ln b_{st})^4 [1-\exp(-\xi_w)]^2}{\Psi_{st}[1-\exp(-\xi_w)]^2}. \tag{13}$$

The term $R_{max}\Psi_{st}$ is eliminated in terms of the burning velocity for a stoichiometric mixture [5, 7]

$$\frac{G Y_{F,w}(1-b_{st})}{[1-\exp(-\xi_w)]} = \{2\rho D \Psi_{st} R_{max} Y_{F,w}(1-b_{st}) \times [1-\exp(-\xi_w)]\}^{1/2} \tag{14}$$

Substituting equation (14) into (13), we get

$$\frac{\dot{m}G}{(4\pi)(\rho D)^2} \approx \frac{b_{st}(\ln b_{st})^2 [1-\exp(-\xi_w)]}{(1-b_{st})[b_{st}-\exp(-\xi_w)]}. \tag{15}$$

Let

$$\frac{\dot{m}G}{(4\pi)(\rho D)^2} = \dot{m}^+ \quad \text{and} \quad Y_{O,\infty}/\varepsilon_F v_s = v. \tag{16a, b}$$

Using equations (8), (10), (14) and (16) in (15), then

$$\dot{m}^+ \approx \frac{1}{v} [\ln(1+v)]^2 / [1 - (\phi_{OF})_{st}]. \tag{17}$$

A plot $\dot{m}^+$ vs v is shown in Fig. 2. From this plot one can estimate the minimum fuel fraction flux for a given injection rate $\dot{m}$ and vice versa. The effects of ambient conditions are contained in the term G. The curve $\dot{m}^+$ reaches a maximum at $v = 3{\cdot}92$. It should also be noted that the minimum injection rate is independent of the injection

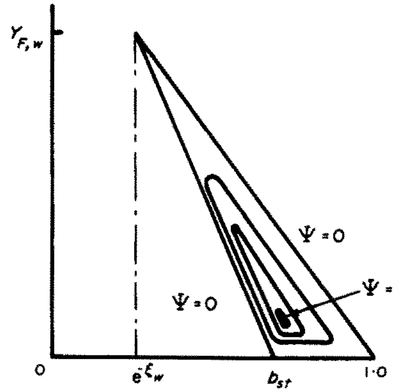

FIG. 1. Spalding's Ψ mountain diagram applied to a spherical diffusion flame maintained by injection of gas.

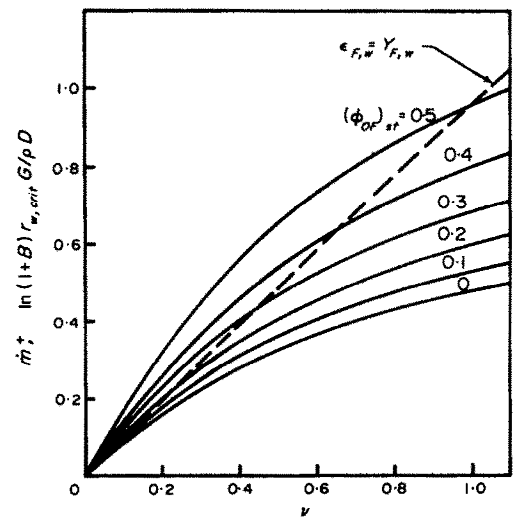

FIG. 2. Minimum dimensionless injection rate vs the stoichiometric parameter; — injection with inert gas, arbitrary ε_F and $Y_{F,w}$; --- injection with inert gas $\varepsilon_F = Y_{F,w}$.

surface geometry and for small values of ν

$$\dot{m}^{+} = \nu/[1-(\phi_{OF})_{st}]. \tag{18}$$

4. RESULTS AND DISCUSSION

In view of the existence of a finite value of extinction injection rate it is apparent that liquid droplets or coal particles with surface areas less than some critical value cannot provide the needed gasification rate to avoid gas phase flame extinction. To estimate the critical particle size the following simplified analysis is made.

Strictly speaking the limiting condition to extinction is a finite kinetic problem. However, one may find a lower bound to the particle surface radius r_w from the results on diffusion controlled burning [8].

$$\frac{[\ln(1+B)]r_{w,\text{crit}}G}{\rho D} = \frac{[\ln(1+\nu)]^2}{\nu[1-(\phi_{OF})_{st}]}. \tag{19}$$

The existance of the critical size for a droplet was shown by Spalding [9] where he deduced this size for a droplet from extension of his one dimensional plane flow theory of maximum flame strength to a spherical diffusion flame. However, the result for critical droplet size in equation (19) agrees with Spalding's result if the stoichiometric parameter ν is very small, in which case $(\phi_{OF})_{st} = 0$, and $\varepsilon_F = Y_{F,w} = 1$. Figure 2 shows a comparison between the present results and Spalding's results (i.e. $(\phi_{OF})_{st} = 0$).

A quantitative estimation is now made of the gas phase extinction injection rate for a butane gas injected through a porous sphere. Since $\dot{m} \propto (\rho D)^2$ the transport properties play a significant role in establishing this estimate. It has been shown by the authors that it is possible to evaluate $(\rho D)_{\text{mean}}$ including concentration and temperature effects for a diffusion controlled spherical flame [10–12]. Thus

$$(\rho D)_{\text{mean}} = 0{\cdot}0033625\,\text{g/cm s}, \quad Y_{O,\infty} = 0{\cdot}23, \quad \nu_s = 3{\cdot}586,$$
$$Y_{F,w} = \varepsilon_F = 1, \quad G = 0{\cdot}04425\,\text{g/cm}^2\,\text{s}\ [13].$$

From equation (17) $\dot{m}^{+} = 0{\cdot}05854$ and hence $\dot{m} = 0{\cdot}000185$ g/s. The authors are aware of only the experimental results of Del Notario *et al.* [6] for this problem. However, they were not able to achieve this limit in view of the fact that the flame moved to the injection surface before this limit of injection was reached. The present results suggest that if inert gas is injected along with the reactant as a mixture, $\dot{m}^{+}$ is increased while G is decreased and consequently $\dot{m}$ is sufficiently increased to perform the experiments with a finite porous sphere.

Similar calculations were performed for gas phase extinction of a highly volatile coal. Using the approach for pyrolysing coal [14]

$$r_{w,\text{crit}} = \left\{ \frac{3\dot{m}^{+}(\rho D)^2}{G\rho_s Y_{v,s} \dfrac{\text{d}}{\text{d}t}\left(\dfrac{\Delta V_v}{V_0}\right)} \right\}^{1/3}$$

it is estimated that gas phase extinction occurs at a size of about 86 μm. Howard [14] estimates the critical size to be about 65 μm by imposing the condition that the flame anchors on the surface, thus assuming *a priori* that the flame continues to survive until it reaches the surface.

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